The Murder-Suicide of the Rentier: Population Aging and the Risk Premium

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Abstract

Population aging has been linked to global declines in real interest rates. A similar trend is seen for equity risk premia, which are on the rise. An existing literature can explain part of the declining trend in safe rates using demographics, but has no mechanism to speak to trends in relative returns on different assets. We calibrate a heterogeneous agent life-cycle model with equity markets and aggregate risk, and we show that aging demographics can simultaneously account for both the majority of a downward trend in the risk free rate, while also increasing the return premium attached to risky assets. This is because the life-cycle savings dynamics that have been well documented exert less pressure on risky assets as older households shift away from risk. Under reasonable calibrations we find declines in the safe rate that are considerably larger than most existing estimates between the years 1990 and 2017. We are also able to account for most of the rise in the equity risk premium. Projecting forward to 2050 we show that persistent demographic forces will continue push the risk free rate further into negative territory, while the equity risk premium remains elevated.

JEL classification codes: E21, E43, G11, J11

Keywords: life-cycle model, OLG model, demographics, rates of return, safe assets, risky assets, secular stagnation.

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1. Introduction

Essential contrasts between different classes of investors and investments have played an important role over the long sweep of macroeconomic thought. In the 19th century, a distinction between a coupon-clipping rentier, vested in fixed-income bonds and debentures, and a risk-taking re-investing capitalist owning mostly private equity, was a crucial one for Marx. Subsequently, in the interwar years of the 20th century Keynes sensed a tectonic shift in the ownership, control, and financing of business, as inflation wracked the traditional rentier class, and a more distributed-ownership public equity model took shape.[1] Looking to the future, and anticipating exogenous technological and political pressures which would force lower real rates of return, Keynes then famously spoke of "the euthanasia of the rentier, of the functionless investor" and thought that, in parallel, even risky returns be would be destined to fall, pari passu, except for some premium "to cover risk and the exercise of skill and judgment."[2] Yet this is not exactly the future that came to pass, either in Keynes's time or our own, even if low real interest rates on safe assets then and now suggest common trends of secular stagnation (Hansen, 1939; Summers, 2014).

The theme of this paper is subtly different and focuses on divergent rates of return. In the last thirty years, real safe rates have indeed fallen, so the classic coupon-clipping rentiers may be seen to have suffered. However, at the same time, returns to risky capital have not fallen as much, if at all. This constitutes a puzzle, and is by definition beyond the scope of standard macroeconomic models with only one type of capital. We introduce an overlapping generations (OLG) model with both safe and risky assets, with labor income and saving for retirement. We study this model under exogenously changing mortality and age structure in recent decades, and look forward to 2050 according to current projections. We have two main findings. First, consistent with current research, our model can generate a decline in the safe rate due to population aging; however, the impact in our model is much larger than the existing literature. Second, we can also match the rise in the risk premium seen in the data, that is, our model also generates a risky return where the change over time is much flatter, resolving the puzzle.

The key mechanism driving our results is endogenous age-specific portfolio choices: workers initially accumulate mainly risky assets ("equity"), but then move to a portfolio with a greater allocation to safe assets ("bonds") as they approach and enter retirement. The model is calibrated to resemble observed patterns in wealth accumulation and portfolio

[1] The idea of the rentier appears in, e.g., Marx (1844 [1932]), Keynes (1936). For critical discussion, see Crotty (1990), and McKibbin (2013). On the evolution of financial systems in those eras see Kindleberger (1984).

[2] The quotes are taken from Chapter 24 of Keynes (1936).

choice in the data. Given this setup, exogenous demographic changes like those seen in the last 30 years produce a growing mass of traditional rentier types in older age cohorts (i.e., boomers) who compete with each other to demand safe assets for their retirement in ever larger numbers, killing the safe rate of return. We therefore describe a phenomenon which is not an exogenous euthansia, but rather an endoegnous murder-suicide of the rentier.

Our work ties in to multiple, large literatures. The past decade has seen advanced economies face a period of unprecedented low real interest rates, stable inflation, and lackluster growth. Recognizing the apparent persistence of these trends, much has been written concerning the secular stagnation hypothesis, repopularized in Summers (2014) and most recently explored in Eggertsson, Lancastre, and Summers (2019a). Indeed a large number of recent works has shown that aging, both through falling fertility rates and rising life expectancy is a significant factor in the long run decline in real rates since as early as the 1980s. One important mechanism linking these demographics to asset prices is the buildup of savings over the course of the lifecycle. This *savings glut* results from a concentration of population into older, higher saving, age groups, as well as increasing life expectancy.

Taking the logic of a savings glut further, one might assume that aging populations have a similar effects on all assets, yet equity returns have stabilized or even risen since 2000. This, coupled with a well documented decline in the risk free rate, has driven a substantial increase in the equity risk premium. We first document this phenomenon empirically, noting that it exists in multiple measurements of the risk premium. We then calibrate a heterogeneous-agent life-cycle model to match the U.S. under different demographic structures, showing that the downward pressure of old age saving on returns is not only weaker on risky assets than on safe assets, but that the effect is quantitatively large.

The literature linking demographic forces to macroeconomic trends has grown substantially in recent years. In their work studying the effect of population aging on real interest rates in the United States, Gagnon, Johannsen, and Lopez-Salido (2016) find that, since 1980, population aging can account for a 125 basis point fall in the long run real interest rates and economic growth. Carvalho, Ferrero, and Nechio (2016) find larger effects, suggesting that the decline may be as high as 200 basis points. They also investigate the strengths of each potential demographic channel and suggest that most of this decline comes from rising life expectancy. While both models differ in some important ways from ours, one of our key mechanisms will be the same. Lisack, Sajedi, and Thwaites (2017) argue that demographic forces can account for roughly half of the roughly 450 basis global real interest rates since 1980 documented in Rachel and Smith (2015), while also accounting for a large fraction of the simultaneous rise in housing prices and debt. Eggertsson, Mehrotra, Singh, and Summers (2016) extend the idea to an open-economy setting showing that global capital markets can act as a transmission mechanism for low natural rates and potential policy spillovers. Recently Eggertsson, Lancastre, and Summers (2019a) show that there are important implications for aging on economic growth and welfare. Cooley and Henriksen (2018) suggest that increasing life expectancies and shifts in cohort distributions can lead to substantially lower growth through decreased labor supply as well as TFP growth. In one of the only other papers to study aging and relative asset prices, Geppert, Ludwig, and Abiry (2016) take a similar modeling approach, but with a focus on projecting forward rather than documenting historical effects. They find much smaller effects on asset prices and our results suggest that much of the important demographic forces on relative asset prices are operating just before the horizon they study.[3]

Empirical work by Daly (2016) suggests that the equity risk premium has risen steadily since the turn of the century. Caballero, Farhi, and Gourinchas (2017) document this for the U.S., finding that from 1980 to 2000, the expected rate of return on equities fell in tandem with risk free rates, at which point equity returns stabilized while safe rates continued to fall and the ERP began to rise. Their conclusion identified the global savings glut as a potential explanation, with a rise in reserve accumulation in emerging markets driving up demand for safe assets relative to risky. Greenwood and Vissing-Jorgensen (2018) find evidence that the size of pension systems, growing with population aging, have driven demand for long term government debt. A relative growth in "risk-averse wealth" is a broader point emphasized by Hall (2016), and it aligns with the mechanism we explore.

We offer a complementary explanation that links this timing seen in the data with the aging of the baby boomers, who in 2000 were 46 to 66 years old. At precisely this time the U.S. workforce began a dramatic shift as this large cohort approached retirement. This trend is also observed in Rachel and Smith (2015), who discuss the growing spread between IMF measures of global real interest rates and their global measure of return on capital.

Our work relates to other research which has identified robust returns or profits in a broader portfolio including risky assets, including observations on flows in the national accounts (Ravikumar, Rupert, and Gomme, 2015) and inferences drawn from summary measures such as $r - g$ (Piketty, 2014). Daly (2016) finds that much of the decline in global bond yields from 1985 to 2000 were driven by this savings channel, but suggests that the majority of subsequent declines come from "equity risk premium shocks." He proposes population aging as a potentially important channel.

[3]Their model includes a few features that ours does not like human capital, but lacks limited participation in asset markets or multiple agent "types" which provide more realistic population asset holdings and wealth. They also project the effects of demographics on growth and safe and risky assets from 2010 to 2050, finding a 0.65 and 0.47 percentage point decline in each respectively. This is smaller than the effect that we find on each asset (2.87 and 3.28 declines), but somewhat similar in the implied ERP change over that period.

The contribution of our work will be to quantitatively examine the effects of aging in a financial model of life-cycle portfolio allocation. Such models have their roots in the seminal work of Bodie, Merton, and Samuelson (1992). They show that agents in a life-cycle model of portfolio choice will rebalance their assets away from risky equity towards safer bonds as they age. The mechanism that is operative in their work, which will be crucial to ours, is that the present value of labor income acts as a relatively safe asset. As individuals age this safe asset shrinks, giving them incentive to move their financial wealth away from risk to keep their overall risk constant. Cocco, Gomes, and Maenhout (2005) build on this result and find that the utility costs of failing to balance portfolio to account for declining human capital assets is potentially quite large. In related work, Benzoni, Collin-Dufresne, and Goldstein (2007) show that equity dividends are cointegrated with the labor markets. The implication of their work is that the expected portfolio shifting takes a hump shape over the life cycle as the human capital aspect of labor income acts as a "stock-like" asset for young investors and a "bond-like" asset for those nearer retirement.

2. The Equity Risk Premium: Recent Trends and Related Evidence

It is useful to define what is meant by the term *equity risk premium* or ERP. In practice there are many ways in which this can be measured and the results are often strikingly different. A useful discussion is presented by Damodaran (2013). Rather than reinvent the wheel, we seek to report a variety of standard estimates which reveal a common underlying pattern. Any measure of the ERP is an attempt to measure the difference between the expected return on a risky asset (R) and the known return on a risk-free asset (R^f), expressed as:

$$RP_t(k) = E_t\left[R_{t+k}\right] - R^f_{t+k}\,.$$

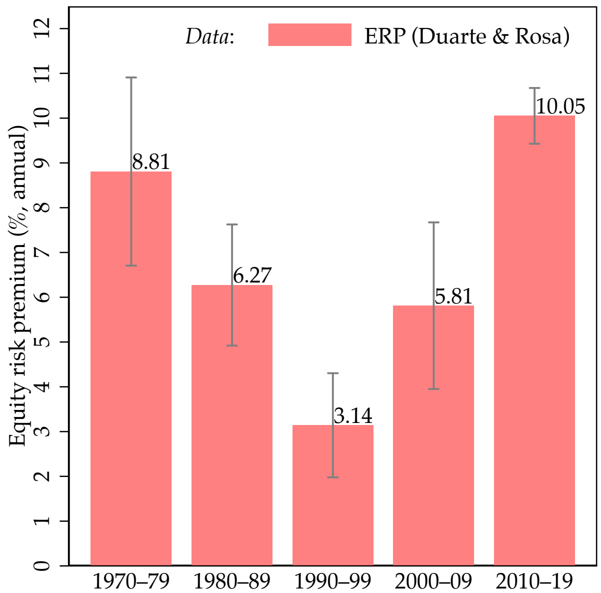
Five broad methods of estimating this object, explained in detail by Duarte and Rosa (2015), are: historical means, dividend discount models, cross-sectional regressions, time-series regressions, and surveys of investors. Among these approaches, the crucial differences largely stem from how expectations are formed, and the specification of equity returns.

Seeking consensus estimates, we follow Duarte and Rosa (2015), and their later updates. They estimated the U.S. equity risk premium for 20 commonly used models over a period from 1960 to 2015. In doing so they both document a wide degree of variance among measures as well as the convergence of those measures over time. Moreover, they conduct a principal component analysis, seeking to extract the common trend in these measures.

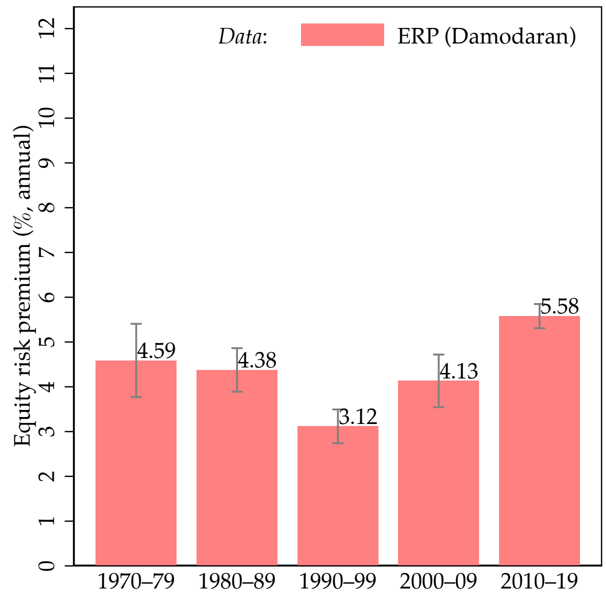
In Figure 1a we see that this ensemble model-averaging approach produces an average annual equity risk premium which declines from near 9% in the 1970s to a low close to

Figure 1: *The Fall and Rise of the US Equity Risk Premium*

(a) *Ensemble principal component (Duarte and Rosa, 2015)*



(b) *Free-cash-flow-to-equity model (Damodaran, 2013)*



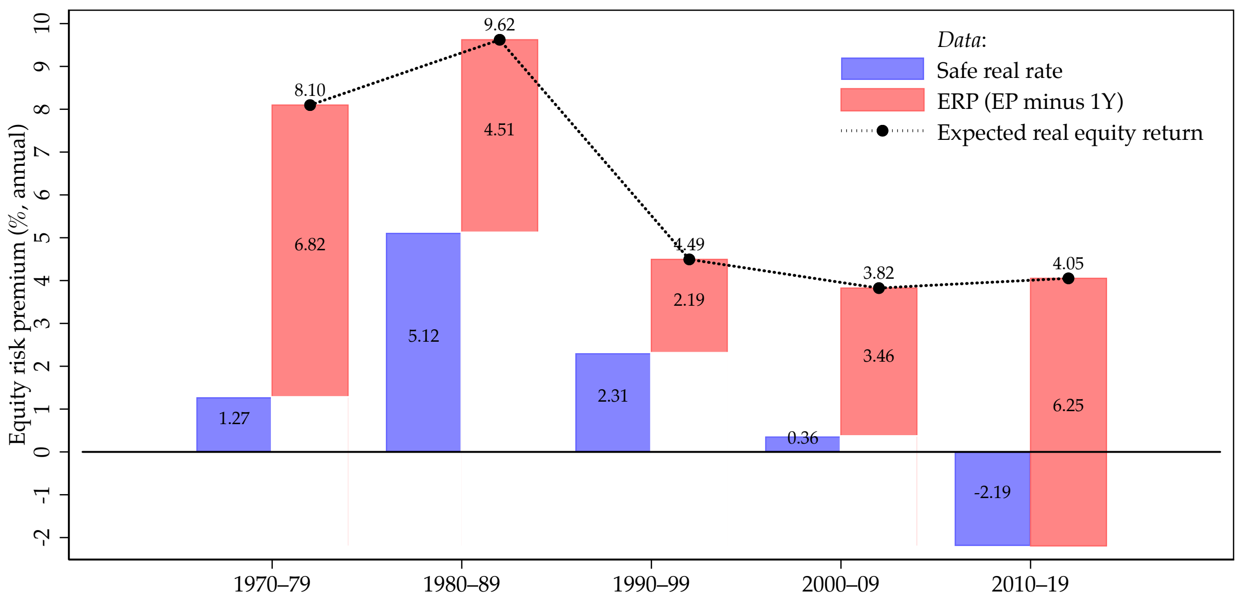
3% in the 1990s, reversing course to rise back to about 10% in the 2010, a swing of roughly 600–700 bps in each direction (95% confidence intervals for each decade are shown).

As there is a great variance in measures of the ERP, we note that similar trends can be found in Figure 1b using the widely-used estimates of Damodaran (2013), and later updates. These are based on a free-cash-flow-to-equity (FCFE) estimation approach: this measure incorporates not only dividends paid, but also the cash left over after taxes, reinvestment, and debt repayments in calculating equity returns. Since only half of this cash has been paid out as dividends over the last decade, and much is used to finance stock buybacks, this measure gives a broad estimate of the return to equity, and could potentially be quite different from the results of some dividend-only models. We see that this model produces a time series for the equity risk premium which declines from near 4.5% in the 1970s to a low close to 3% in the 1990s, then reversing course to rise back to about 5.5% in the 2010.

Qualitatively this again reveals a U-shaped path over time, as in the ensemble model; but quantitatively the variation is smaller, and the amplitude of the swings here are less pronounced, around 150–250 bps rather than 600–700 bps. Given the amplitude of the changes seen, we treat the ensemble model principal component as a "high" estimate of the decadal ERP variation to be explained, and the FCFE model as a "low" estimate.

Next, we present a compromise "medium" estimate of decadal ERP changes in Figure 2. We plot ten year averages for the expected real return on equity, the real safe rate,

Figure 2: *Safe rate, expected equity return, and risk premium using Shiller E/P and 1-year treasury yield*



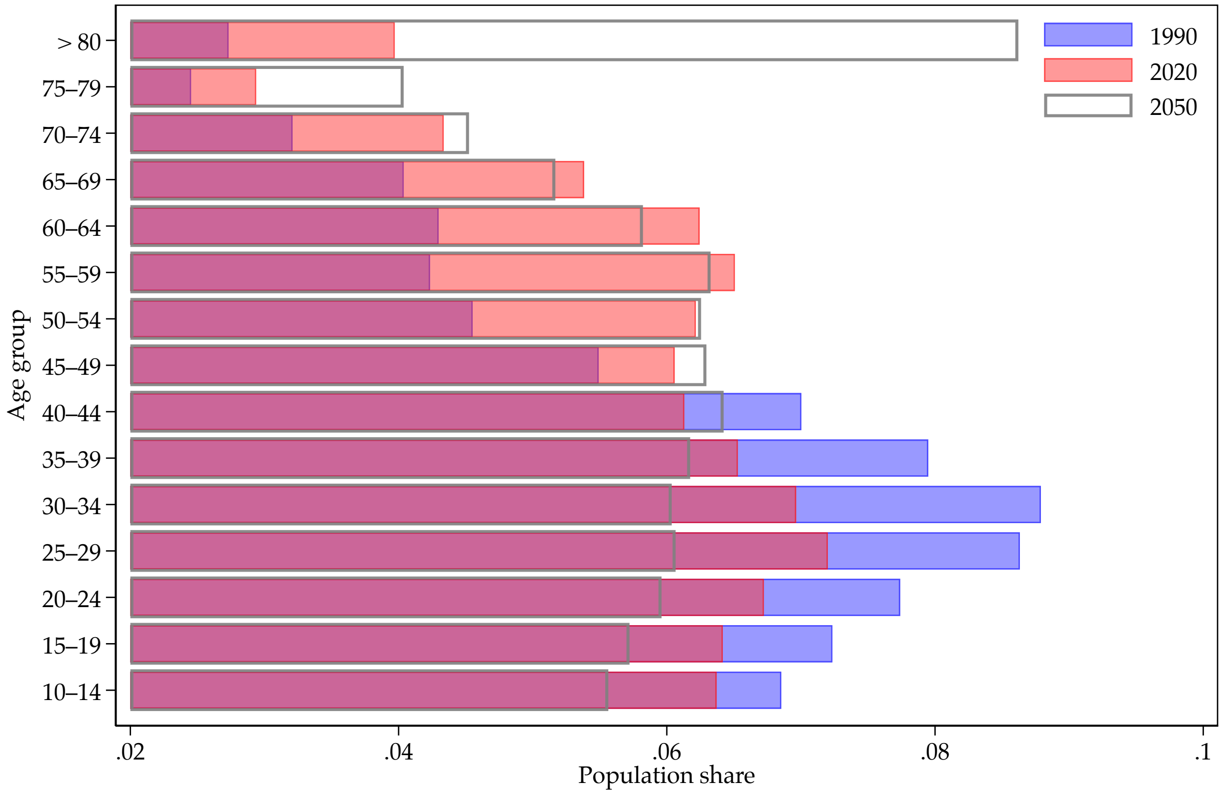
and the implied equity risk premium. Here the expected return on equity is the inverse Shiller price-earnings ratios and the safe rate is the 1-year treasury bill yield. Expected inflation uses the Michigan survey, or when that is not available, lagged moving-average CPI inflation.

Looking at this picture, the average expected equity return was elevated in the 1970s and 1980s, but relatively stable since the 1990s at around 400 bps, and the slight dip seen in the 2000s is in line with the findings of Caballero, Farhi, and Gourinchas (2017) and others. On the other hand, the average real safe rate shows the steady downtrend from 500 bps in the 1980s to near zero as is well known from the natural rate and secular stagnation literatures (Holston, Laubach, and Williams, 2017; Summers and Rachel, 2019). Thus, the average risk premium fell from high levels of 450–680 bps in the 1970s and 1980s, to a low of near 220 bps in the 1990s, only to widen back to near 350 bps in the 2000s and 625 bps by the 2010s.

Summing up, by any measure the risk premium fell from the 1970s to the 1990s, then rebounded substantially over the last 20–30 years, as seen in the last two sets of estimates. This is qualitatively in line with the trends that Duarte and Rosa (2015) derive from 20 different measures of the ERP, so this stylized fact is not an artifact of one choice of measurement of equity returns. Since it represents a "medium" amplitude estimate of low-frequency decadal ERP variations, we treat Figure 2 as our explicandum for the rest of this paper. Can a macroeconomic model explain a meaningful share of these patterns?

Our model builds on demographic change. The United States is still in the midst of a

Figure 3: *U.S. changing age cohort sizes*

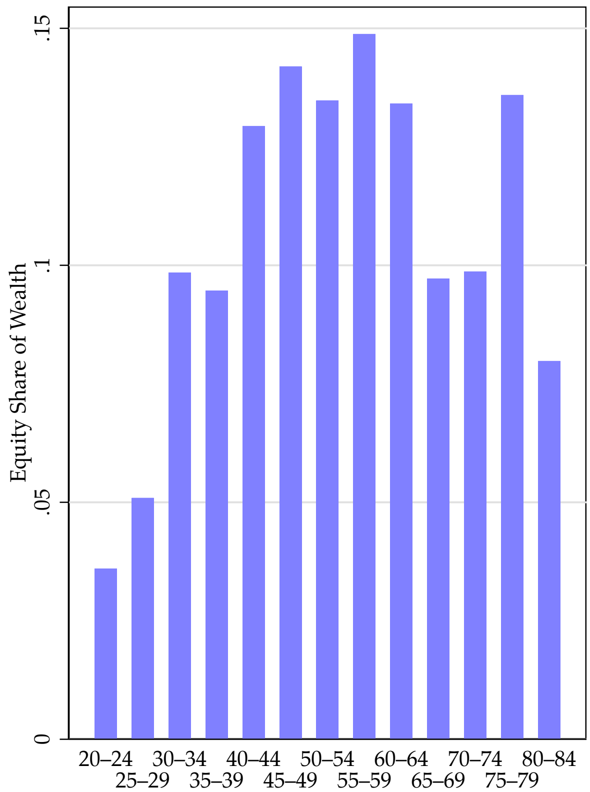


long trend in population aging. Figure 3 shows five year cohort shares of overall population in 1990, 2020, and then projections for 2050. There has been a striking change in the age structure of the population since 1990 with no end in sight. Any demographic headwinds or tailwinds acting on asset prices will be with us for a long time.

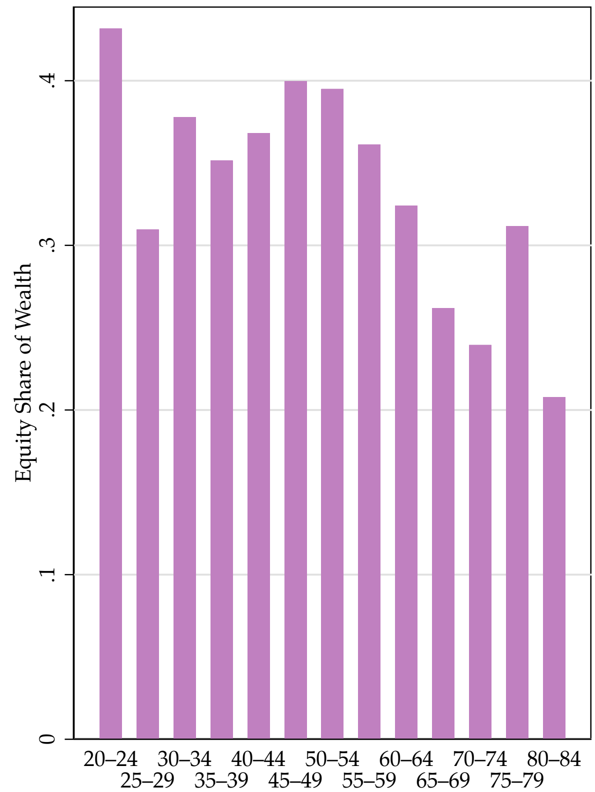
Our model then adds endogenous asset allocation over the course of working life and retirement. As noted, standard life-cycle models of portfolio allocation find that households will rebalance their financial portfolios away from risk as they approach retirement. In much of the literature, this hypothesis has found mixed empirical support. However, in work with relatively complete administrative data on asset holdings such as Fagereng, Gottlieb, and Guiso (2017) it seems true that households significantly draw down equity assets as they age. They show that around retirement households not only reduce their portfolio share of equity, but also start to leave the stock market altogether. They show that to achieve change on both of these margins in a partial equilibrium life-cycle model of portfolio choice there must be a participation cost as well as large downside risk that forces households out of the market under certain wealth thresholds.

To explore this idea in a U.S. context, Figure 4 shows shares of wealth by five year age

Figure 4: *U.S. Survey of Consumer Finances: Equity portfolio share by age*



(a) *Full sample*



(b) *Stock market participants*

groups for the U.S. from the 2016 Survey of Consumer Finances (SCF). To generate these figures we first add up all financial wealth of a household and take the ratio of equity holdings to the total portfolio. This requires a few assumptions. We assumed that stocks comprise 60% of "combination" funds and ETFs in the SCF. This choice has little effect on the overall estimates. Second, we include the value of pension and social security income as an asset for retired households. To do this we use existing data on pensions and social security benefits and assume a discount factor of 0.97. Using this, along with a household's remaining years of life expectancy we calculate the net present value of pensions and social security assets and include them in this measure of financial wealth. Adjusting wealth in this way is important to consider because these social security payments will be important for the financial decisions made in the model.

We will not try to calibrate with the goal of matching these portfolio shares (a tall order in such models). Like any typical macro life-cycle models we will also find it hard to avoid counterfactually high equity participation, particularly for early age groups where it is challenging to make them hold less than 100% equity. However, the measure we construct here is, we feel, a more apt comparison to our model outcomes which we hope to qualitatively match and illustrates motivation that the life-cycle mechanisms driving our results are substantial in the data.

3. Model

3.1. Environment

Our model closely resembles Gomes and Michaelides (2008). They are one of a small, but growing, literature that allows for aggregate risk in an incomplete market life-cycle framework.

In particular, to understand the role of demographics on relative asset prices we need: equity portfolio choice, life-cycle structure, and household heterogeneity. Households have finite lives that are divided exogenously between working life and retirement. We focus our current analysis on stationary equilibria under various demographic structures.

It would be ideal rather to study the transition of this model economy from one demographic structure to another. However, this would require aggregate uncertainty along this transition path which would be computationally challenging. Much of the literature that studies life-cycle transitions in this context rely on perfect foresight equilibriums to make this problem tractable.

In their working life, households earn wage income that is subject to idiosyncratic shocks. These households have access to two investment assets. The first is a riskless government bond, and the second is an equity asset that taking the form of claims on a risky capital stock. In addition, we follow both Gomes and Michaelides (2008), as well as Fagereng, Gottlieb, and Guiso (2017), in requiring that households must pay a fixed participation cost in order to be able to participate in these equity markets. For simplicity we require that this cost only be paid once upon the first access to these markets. The retirement age is exogenously fixed at $R = 65$.

Perfectly competitive firms produce the consumption good using capital and labor in a constant returns to scale technology. There is a government sector that runs a social security scheme that is financed through taxes on wages, while also financing government expenditures and debt interest payments through taxes on capital gains.

3.2. Production

Technology is characterized by a Cobb-Douglas production function with total output at time t given by:

$$Y_t = Z_t K_t^{\alpha} L_t^{1-\alpha} ,$$

where K is the total capital stock in the economy, L_t is the total labor supply, and Z_t a stochastic productivity shock, which follows the following process:

$$Z_t = G_t U_t\,,$$
$$G_t = (1+g)^t\,.$$

The variable U_t represents productivity shocks that follow a two-state Markov chain and matches the average business-cycle duration. Exogenous secular growth is determined by g. After observing the aggregate shock, firms make decisions. With δ the depreciation rate of capital, factor prices can be determined by the firms profit maximization problem as:

$$W_t = (1-\alpha) Z_t \frac{K_t}{L_t}^{\alpha}\,,$$
$$R_t^K = \alpha Z_t \left(\frac{L_t}{K_t}\right)^{(1-\alpha)} - \delta_t\,.$$

To help generate return volatility we also include a stochastic depreciation rate. This allows us to generate similar effects to adjustment costs while sidestepping complications that would arise in an incomplete markets model. This is used extensively in this literature and is given by:

$$\delta_t = \delta + \varsigma \eta_t\,,$$

where η_t is a standard normal shock and ς is a scaling parameter. This depreciation shock is uncorrelated with our productivity shocks.

3.3. Government

Social security is commonly used in life-cycle models as a means of generating realistic labor income processes. In our case it is also crucial in that it has meaningful impacts on the stockpiling, and drawing down, of wealth as households age.

A specification of this model without social security payments would have the effect of increasing the age-specific risk households are exposed to and exaggerate the mechanism that delivers our results. The government is also responsible for supplying the risk free assets to households. We will follow Gomes and Michaelides (2008) and model the government sector as supplying a positive net supply of bonds. It would be difficult to match portfolios found in data if the government is restricted to a zero net supply of bonds in a way that is more common in the life-cycle literature, while modeling an endogenous government supply is beyond the scope of our work and might muddy the effect of demographics.

We assume

$$SS_t^{pay} + G_t + R_t^B B_t = B_{t+1} - B_t + T_t + SS_t^{rev}\,, \tag{1}$$

where G is government consumption, B government debt, R^B the interest rate on government bonds, T tax revenues from non social security taxes, and the social security payments and revenues are given by the SS terms, which are separated here because the system is always in balance and they drop out of this budget constraint. The social security system is funded through taxes on labor income, τ_{ss}, and payments to retired individuals are given as a fraction of their lifetime earnings, λ_{ss}.

Note that our model will abstract from problems of social security imbalance that are both critical to the actual situation of the United States, and also possibly a channel that could be important to the long run implications of the model. Kitao (2014) and İmrohoroğlu and Kitao (2009) provide an excellent reference for how social security operates in life-cycle models in general, the former giving an in depth exploration of the various mechanisms that can solve these imbalances.

3.4. Financial markets

Households have access to two financial assets, a one-period riskless asset and a risky investment opportunity. Agents buy the risk-free asset for price P_t^b, which returns one unit of the consumption good in the following period. Thus,

$$R_t^b = \frac{1}{P_{t-1}^B} - 1\,.$$

The return on the risky asset is denoted by R_t^K. Additionally, investors must pay a one time fixed cost, F the first time they invest in equity markets.

3.5. Households

3.5.1 Preferences and Labor

The household sector is populated by ex-ante identical individuals, facing finite and uncertain lives. In order to generate sufficiently large risk premiums, we adopt dynamic preferences developed by Epstein and Zin (2013). Given that ρ_i is the coefficient of relative risk aversion (CRRA), ψ_i is the elasticity of intertemporal substitution (EIS), and β is the discount factor, these preferences at age, a, can be defined as:

$$V_a = \left\{ (1-\beta) C_a^{1-\frac{1}{\psi_i}} + \beta \left(\mathbb{E}_a \left[s_{a,i} V_{a+1}^{1-\rho_i} \right]^{\frac{1-1/\psi_i}{1-\rho_i}} \right) \right\}^{\frac{1}{1-1/\psi_i}} .$$

There are two agent types, differing based on both their relative risk aversion ρ_i as well as their EIS, ψ_i. With heterogeneity in both their risk aversion and willingness to substitute consumption inter-temporally. In our baseline specification of the model conditional annual survival probabilities, $s_{a,i}$ will be uniform across these two types such that the demographic forces act equally on both groups.

In the baseline specification all households supply labor inelastically. The labor income of individual, i, follows a stocastic process such that their labor income is given by $W_t \ell_{a,t}$. Where W_t is the aggregate wage and $\ell_{a,t}$ is the idiosyncratic and permanent random components to their wages:

$$\ell^i = \exp\left(\xi_a^i\right) N_a^i . \tag{2}$$

The term N_a^i represents the household's permanent idiosyncratic wage shock, which contains a deterministic age-specific trend n_a, and ξ_a^i is a transitory shock. The two shocks can be described as follows.

$$N_a^i = N_{a-1}^i exp(n_a \varepsilon_a^{\epsilon}) \; ; \quad \ln \varepsilon_a^{\epsilon} \sim N(0, \sigma_{\epsilon}^2) \; ; \quad \ln \xi_a^i \sim N(0, \sigma_{\epsilon}^2) . \tag{3}$$

3.5.2 Demographics

Individuals live for a maximum of N periods, with conditional survival probability for individuals aged a in period t given by: $s_{a,t}$. This is the probability that an individual lives to age a+1 conditional on having reached age a. Thus $1 - s_a$ represents the probability that an individual will die before moving to the next period. If life expectancy falls this will appear in the conditional mortality and individuals will more heavily discount the future retirement due to the lowered expectations that they will survive to enjoy consumption in later periods.

In addition fertility rates change every year, and the fertility of an age group. The relative size of cohorts are a function of both the fertility rates of the cohort as well as survival probabilities. Simply put each cohort is born a certain size and dies off at a certain rate. In the present specification of the model not only are demographics exogenous, but since we run simulations for households in a fixed demographic period without simulating the transition we do not actually simulate a change in the underlying population, rather keeping population weights fixed in their respective years.

The Human Mortality Database (2019), provide information both on age-specific mortality as well as information to calculate age-specific population weights. For future population projections we use five year United Nations population projection data to generate these weights. We denote cohort sizes in a given period as χ_t, which is a vector containing individuals of every age group at time t.

3.5.3 Household Wealth

Total liquid wealth can be consumed or invested in these two assets. Denote household wealth as cash-on-hand $X_{a,t}$ and an indicator I_p to denote as 1 if the individual has not yet paid the participation cost and zero otherwise. Then denote the wealth of a working individual age a and time t as:

$$X^i_{a,t} = K^i_{a,t}(1 + (1 - \tau_K)R^K_t) + B^i_{a,t}(1 + (1 - \tau_K)R^B_t) + \ell^i_{a,t}W_t - I^iF\,. \tag{4}$$

After retirement, individuals wage income is replaced by the social security income, given by a fraction of their wage income at retirement. Additionally households are not able to borrow against future labor income, and cannot short any asset. During retirement years $(a > R)$, household's cash-on-hand is given by:

$$X^i_{a,t} = K^i_{a,t}(1 + (1 - \tau_K)R^K_t) + B^i_{a,t}(1 + (1 - \tau_K)R^B_t) + \lambda_{ss}\ell^i_{a,R}(1 - \tau_{ss})W_t - I^iF\,. \tag{5}$$

Inability to borrow against future income or short assets are represented with the following constraints:

$$B^i_{a,t} \geq 0\,, \quad K^i_{a,t} \geq 0\,.$$

3.6. Individual Optimization

Households take prices as given and maximize utility of consumption and leisure given expectations about future aggregate wages and asset returns. A rational expectations equilibrium requires that agents accurately predict the values for wages and rental rates. In heterogeneous models without aggregate risk of the type of Aiyagari (1994) this is not a problem as mean zero idiosyncratic risk does not affect aggregate wages and rental rates. Since labor supply and capital stock are endogenous to household investment decisions in the presence of risky equity, we must an algorithm similar to Krusell and Smith (1998).

The household optimization problem needs to include state variables that allow agents to forecast values for K_t and P^B_t. While doing so exactly requires the infinite-dimensional wealth distribution, Krusell and Smith (1998) show that it is possible to approximate this with a small set of moments. This can be accomplished in this context using lagged values of aggregate variables K_t and P^B_{t+1} as well as realizations of the aggregate shock, U_t and the stochastic depreciation η_t. These variables must now be state variables in the household value function:

$$\begin{aligned} K_{t+1} &= \Gamma^K(K_t, P^B_t, U_t, \eta_{t+1})\,, \\ P^B_{t+1} &= \Gamma^L(K_t, P^B_t, U_t, \eta_{t+1})\,. \end{aligned} \tag{6}$$

3.7. Solving the household's problem

The individual's problem is solved for a stationary equilibrium where individual variables are normalized to the permanent component of household labor income $N_a(G^{\frac{1}{1-\alpha}})$ and aggregate variables normalized by aggregate productivity growth $(G_t^{\frac{1}{1-\alpha}})$. Normalized variables are denoted by lowercase letters. The problem is:

$$V_a\left(x^i_{a,t}, I^i_p; k_t, l_t, U_t, P^B_t\right) = \max_{c^i_a, h^i_a, k^i_{a+1}, b^i_{a+1}} u_a(c_a, h_a) \;+\; \beta s_i \mathbb{E}_{a,t}\left[((N^i_{a+1}/N^i_a)(G^{\frac{1}{1-\alpha}}))^{-1} V_{a+1}\left(x^i_{a+1,t+1}, I^i_p; k_{t+1}, l_{t+1}, U_{t+1}\right)\right]\,,$$

subject to:

$$\begin{aligned} k^i_{a+1,t+1} &\geq 0\,, \\ b^i_{a+1,t+1} &\geq 0\,, \\ x^i_{a,t} &= k^i_{a+1,t+1} + b^i_{a+1,t+1} + c^i_{a,t}\,, \\ x^i_{a+1,t+1} &= \frac{k^i_{a+1,t+1}(1+R^K_{t+1}) + b^i_{a+1,t+1}(1+R^B_{t+1})}{(N^i_{a+1}/N^i_a)(G^{\frac{1}{1-\alpha}})} \;+\; w_t e^{\epsilon_i} \;-\; I^i_p F\,, \\ R^K_{t+1} &= R(k_{t+1}, U_{t+1})\,, \\ w_{t+1} &= W(k_{t+1}, U_{t+1})\,, \\ k_{t+1} &= \Gamma^K(k_t, P^B_t, U_t, \eta_t)\,, \\ P^B_{t+1} &= \Gamma^L(k_t, P^B_t, U_t, \eta_t)\,. \end{aligned} \tag{7}$$

3.8. Equilibrium

A steady-state equilibrium is a set of endogenously determined prices, value functions, and policy rules that are specific to age cohorts, and rational expectations by individual agents over the evolution of all endogenously determined variables. We then have:

Households Optimize: Households follow cohort specific policy rules: $\{V_a, b_a, k_a\}_{a=1}^{N}$ are consistent with their dynamic programming problem given by Equation 7.

Firm Optimize: Firms maximize profits by setting their MPK and MPL equal to their marginal costs R_t and W_t.

Markets Clear: Such that aggregates are equal to the sum of individual decisions:

$$\begin{aligned} K_t &= \int_i \int_a N_{a-1}\, k^i_{a,t}\, \chi_i \, da \, di \,, \\ B_t &= \int_i \int_a N_{a-1}\, b^i_{a,t}\, \chi_i \, da \, di \,, \\ L_t &= \int_i \int_a N_{a-1}\, \ell^i_{a,t}\, \chi_i \, da \, di \,, \\ U_t\, K_t^{\alpha}\, L_t^{1-\alpha} &= \frac{C_t^G}{G_t^{\frac{1}{1-\alpha}}} + (1+g)^{\frac{1}{1-\alpha}} K_t - (1-\delta) K_t + \int_i \int_a P^i_a\, c^i_{a,t}\, \chi_i \, da \, di \,. \end{aligned} \tag{8}$$

Government Balances: Both its own budget constraint each period, maintaining a given level of debt to GDP, as well the social security system at all times:

$$\int_i \int_{a=0}^{a=R} \tau_{ss}\, l^i_a\, \chi_i \, da \, di = \int_i \int_{a=R}^{a=N} \lambda_{ss}\, \ell_{R,t}\, W_t\, \chi_i \, da \, di \,. \tag{9}$$

Prices: Are verified in equilibrium.

Analytical solutions are not possible in in this model. In the following section we sketch the solution method to solve for a stationary equilibrium computationally in this model.

3.9. Solution Method

1. Specify forecasting equations: Γ^K and Γ^{P^B}.

2. Solve the household's decisions problem taking prices as given and using forecasting equations to form expectations. All state variables are mapped into a discrete state space and optimal policy rules are solved by backwards induction from the final year of life.

3. Given policy functions in part 2, simulate the model (2000 periods). Check market clearing conditions.

4. Use the simulated time series to update forecasting equations

5. Repeat 2–4 until convergence:

   - Markets must clear within tolerance;
   - Stable coefficients in the forecasting equations;
   - Forecasting with regression R^2 above 99%.

3.10. Simulation

Realizations of the aggregate random shock are drawn from its two state Markov distribution and individual agents decisions are simulated conditional on their individual draws from the log-normal productivity shock.

For each time period household behavior is simulated for every possible bond price. Then individual demands are aggregated and linear interpolation is used to determine the market clearing bond price. This determines simulated state variables for next period decisions and the process is repeated. We simulate the long run steady state of the economy under each demographic regime. While this abstracts from the real world transition dynamics allows for comparison of demographic effects in a way that is computationally much less burdensome.

3.11. Updating the forecasting equations

Using the simulated time series, forecasting equations are estimated using OLS regressions. For each realization of the productivity shock, U_t and given known change in the employment population ratio, λ_{t+1} we simulate the following:

$$\begin{aligned} ln(k_{t+1}) &= \beta_{1,0} + \beta_{1,1} ln(k_t) \,, \\ ln(P^B_{t+1}) &= \beta_{2,0} + \beta_{2,1} ln(k_t) + \beta_{2,2} P^B_t \,. \end{aligned} \tag{10}$$

Which for the baseline specification yields eight equations with separate coefficients to be estimated. Convergence of our simulation requires both that the R^2 of each of these forecasting equations is greater than 99% under each set of aggregate states and that all coefficients converge.

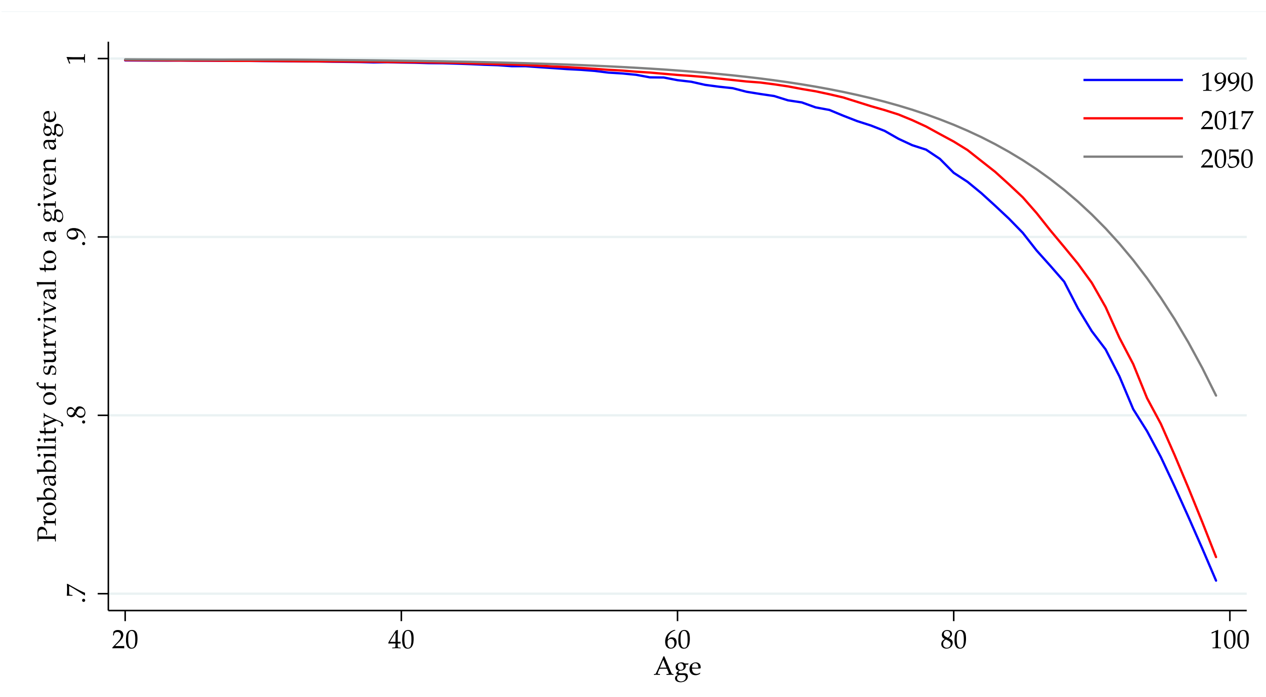
4. Calibration

4.1. Demographics

Our key dimension of analysis is changing population structure. We take retirement age as exogenous, fixed at $R = 65$. Annual conditional mortality rates used in our model are shown in Figure 5. Conditional annual survival probabilities could be calculated using the method described in Henriksen (2015) in 2050, while those for 1990 and 2017 come from the Human Mortality Database (2019), henceforth HMD. For demographic weights we use the HMD for 1990 and 2017 while using the United Nations data from Figure 3 interpolated to annual frequency.

For our quantitative estimation we calculate the general equilibrium conditional on the demographic state in a given year as if it were fixed permanently. Implicitly this assumes that individuals in our economy believe that the current demographic structure, and any impact it has on prices, will persist into the future. Given that we are interested in long run effects, and that expectations by investors regarding the effect of demographics on prices in the future should be a second order effect we think it is reasonably harmless to make this simplification. Quantitatively estimating the model along the entire transition path would be computationally burdensome and pose a set of challenges without, in our estimation,

Figure 5: *Survival probabilities by age*



providing particularly insightful results relative to our current approach.

We will show results for the model calibrated to five "steady state" demographic structures in: 1970, 1990, 2010, 2017, and 2050. These reflect the age structure and life expectancy (through age specific mortality) in these years. The results in 1970 provide something of a pre-boomer baseline as the oldest members of that cohort will be 24 and relatively small players in asset markets. Our focus will be on results from 1990 to 2017, as well as forecasts to 2050. These three are particularly important because 1990 represents an early year where the entire baby boomer cohort is participating in the labor force (at this point the youngest boomer is 26 years old). In 2017 boomers were at peak savings age with the youngest at age 53 and the oldest 71. By 2050 the age range of baby boomers is 86–104, almost completely aged out of the model, and the relatively "flat" age structure in Figure 3 should not see dramatic change.

We next describe the household, production and government calibration, with details shown in Table 1.

4.2. Household Variables

There are two agent *types* in the economy who differ along two dimensions. The first type has low risk aversion, with CRRA $\rho_A = 1.1$ and low elasticity of inter-temporal substitution, with EIS $\psi_A = 0.0833$. The second agent has higher CRRA, with $\rho_B = 5$, as well as higher EIS, with $\psi_B = 0.3$. Both agents have the same discount factor $\beta = 0.99$. Giving type A agents low risk aversion causes a reduction in early life savings due to lowered precautionary motive. A low EIS also limits life-cycle savings to smooth consumption. They thus accumulate relatively less in mid-life in preparation for retirement. These effects induce them to endogenously accumulate little in the way of savings, with the poorer members of this type being completely "hand-to-mouth". For those with savings a fixed participation cost induces them to rarely hold equity. Type B agents will endogenously act as the major participants in both asset markets and, in most specifications, are the only holders of equity. We assume an equal share of these agent types for simplicity.

We believe these are reasonable parameters. Guvenen (2006) shows that limited stock market participation along with EZ preferences can reconcile disagreement with the macro literature on EIS parameters. This disagreement stems from micro consumption data often implying EIS close to zero and macro correlations implying a value close to 1. Models with limited participation can remedy this by separating the effect of the average consumer from that of the average investor. Both of our preference parameters for the EIS (0.0833 and 0.3) can be reasonably supported in the empirical literature. In their study of preferred consumption paths Barsky, Juster, Kimball, and Shapiro (1997) find a lower bound of the

Table 1: *Calibration*

Household		
β	Time discount rate	0.99
ρ^A	Risk aversion type-A	1.1
ρ^B	Risk aversion type-B	5
ψ^A	EIS type-A	0.0833
ψ^B	EIS type-B	0.30
n_a	Age-specific trend parameters	{ 0.174 , -0.237 , 0.00611 }
s_i	Age-specific mortality rate	HMD or UN Forecasts
χ_i	Cohort size	HMD or UN Forecasts
Production		
α	Capital share	0.36
δ	Depreciation	0.10
σ_U	Volatility of aggregate productivity	0.01
ς	Depreciation shock (scaling std. normal)	0.15
Π	Aggregate shock process	$\begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix}$
Government		
B^S	Supply of government bonds, fraction of output	0.30
λ_{ss}	Social security replacement rate	0.40

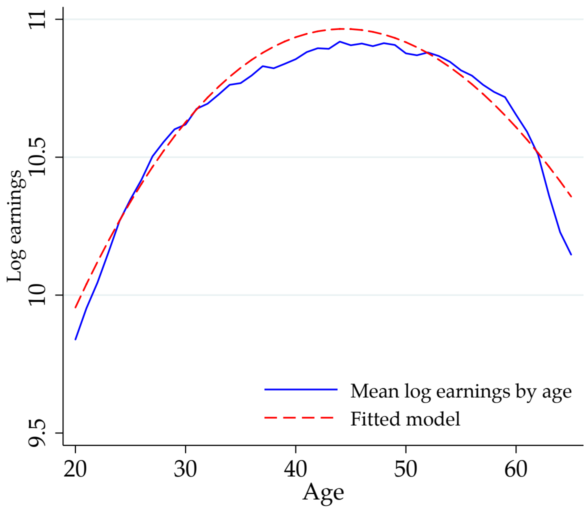
EIS of close to zero and an average upper bound of roughly 0.36 with a mean of 0.18. The average in our economy under our preferred specification is 0.19. In their estimation of risk parameters they find a mean risk tolerance of 0.24. The reciprocal of this, 4.2 is the harmonic mean risk aversion in their sample. They find the arithmetic mean is quite different at 12.1. We think our choice of $\rho^B = 5$ for stockholders is a reasonable one given this. As our average across all households of roughly 3 is closer to the micro evidence than those commonly used in macro modeling.

The household earnings process has an age-specific trend, n_a, which is calibrated to match the average for the PSID similar to Cocco, Gomes, and Maenhout (2005), Huggett and Kaplan (2016), and others. We fit a third order polynomial on the life-cycle income process with year fixed effects. Figure 6 shows both the sample mean of log earnings over the working life as well as our fitted model. The volatility of idiosyncratic income shocks are set to 10% per year, in line with estimates used in Cocco, Gomes, and Maenhout (2005).

The fixed cost of participation is set such that it corresponds to 7.5% the household's expected annual income. Moskowitz and Vissing-Jørgensen (2002) suggests a per-period costs that are approximately \$75–\$200 each year. While this one time fixed cost is quite

Figure 6: *Age-specific wage calibration*

(a) *Earnings over working life*



(b) *Fitted model*

	(1) log earnings
age	0.174***
	(0.0141)
$\frac{age^2}{100}$	-0.237***
	(0.0359)
$\frac{age^3}{1000}$	0.00611*
	(0.00291)
Year FE	no
R^2	0.103
N	80273

Standard errors in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

large, it is similar to those and other estimates of participation costs in present value terms. It can be adjusted, or even set to zero with little affect on overall results if the relative preference parameters of the two agent types are adjusted accordingly.

4.3. Aggregate and government variables

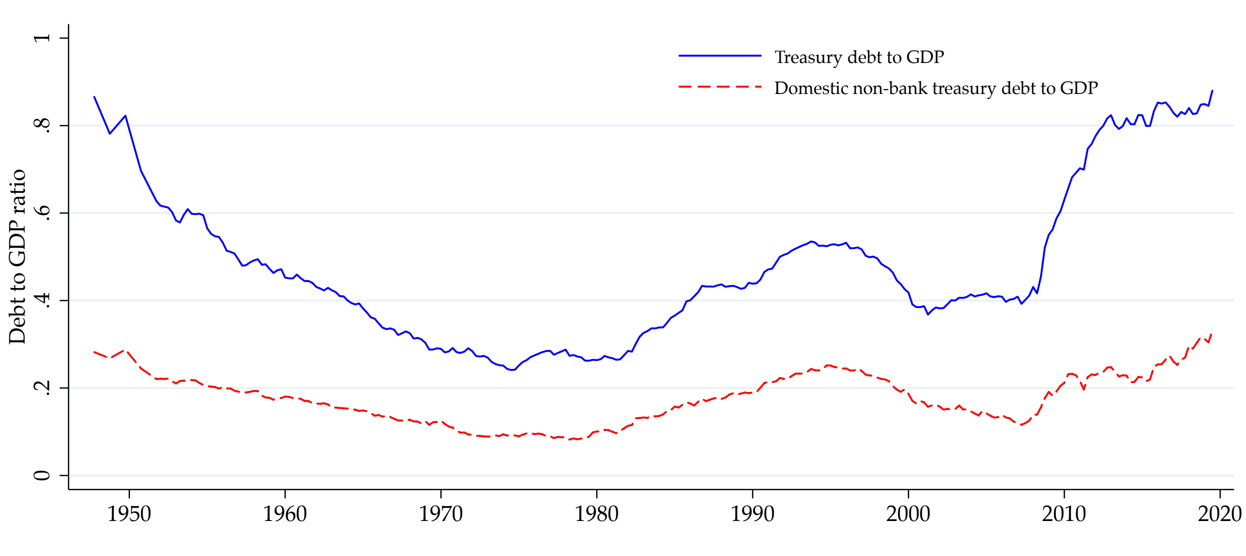
In order to match a business cycle duration that is on average six years, the two state Markov process for aggregate productivity is given by

$$\Pi = \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix}. \tag{11}$$

The volatility of aggregate productivity, (σ_u), is set at 1%. Capital's share of output is set to 36%, and depreciation is 10%. The volatility of asset returns is predominantly determined by ς, which is set at 15%.

We set the net positive supply of government bonds to be a constant share of GDP. While government debt has risen substantially between over the period of study, the share held by the domestic non-bank public has remained relatively low and has seen substantially less variation. We calibrate this using Treasury debt held by the domestic non-bank investors using data from the Federal Reserve, Financial Accounts of the United States. We sum all debt held by: households, government retirement accounts, private pension funds, money market funds, mutual funds, ETFs, and closed end funds.

Figure 7: *Government debt to GDP*



These are graphed in Figure 7 along with the overall level of treasury debt to GDP. The larger trend in debt comes mainly from holdings by the central bank, financial institutions, and foreign holders. For our benchmark model we set this at 30% of equilibrium GDP, and hold it constant over time. While a fixed bond supply is a strong assumption, we feel it is the only way to understand the impact of demographics directly. There will be some movement in bond supply, but only through demographic effects on output.

The retirement social security transfer: λ_{ss} is set at 0.4 using the same parameter as the benchmark model in Kitao (2014) roughly the average benefits over average earnings. The social security tax is set to clear the governments requirement to balance the social security budget. This comes to about 15%, but increases as the employment population ratio shrinks. If we instead allowed the government to issue more debt to finance social security into the future without fully funding it our results would likely be amplified due to an increased supply of the safe asset. We abstract from bequests by assuming that the government taxes assets at 100% upon death.

5. Results

We now describe the results that come from this model. We solve and simulate the model under a number of different demographic structures. As mentioned these represent a steady state where the demographic structure is stable. This is an abstraction from the reality in a potentially problematic way, but solving this model for transition dynamics between two steady states is computationally intractable. However, we find it useful to see the degree to which the risk premium is altered in these different steady states.

In Figure 8, show the average cash-on-hand wealth in the model simulations under the 1990 and 2017 demographic structures. We see that overall households are saving more as life expectancies are rising, with some slight change in the curvature. This is because there are significant increases in some of the survival probabilities for retirement aged individuals between these two periods, so the draw down of wealth is smoothed relative to the 1990 case. The general "hump" shape reflects the accumulation of wealth as individuals both buffer against idiosyncratic shocks, as well as building up savings that can then be drawn down in retirement.

Figure 9 shows a striking result. Average stock and bond holdings across ages differ dramatically between these two demographic structures. The life-cycle preference to hold more safe assets is quite strong in the periods just before, and in, retirement. We see that older age households now hold fewer safe assets relative to equity in 2017. This should work against our expected effect of rising equity risk premia, but is also not terribly surprising. As the safe rate is driven down (as we shall see), with large fractions of households in old age, general equilibrium effects will cause households to endogenously choose to hold higher fractions of relatively more attractive equity. Given we don't change the riskiness of equity between the two periods, it is natural that these equilibrium forces would work in this direction. Rising life expectancy likely mutes some of this effect as households desire to self-finance part of their retirement income, a longer expected period in 2017 than in 1990.

Figure 10 shows the share of equity in the financial wealth of individuals. To improve readability we omit the first five years of working life where these start at zero and then rapidly increase, as well as the last possible year of life where death is certain and remaining

Figure 8: *Financial wealth in the model: total wealth by age*

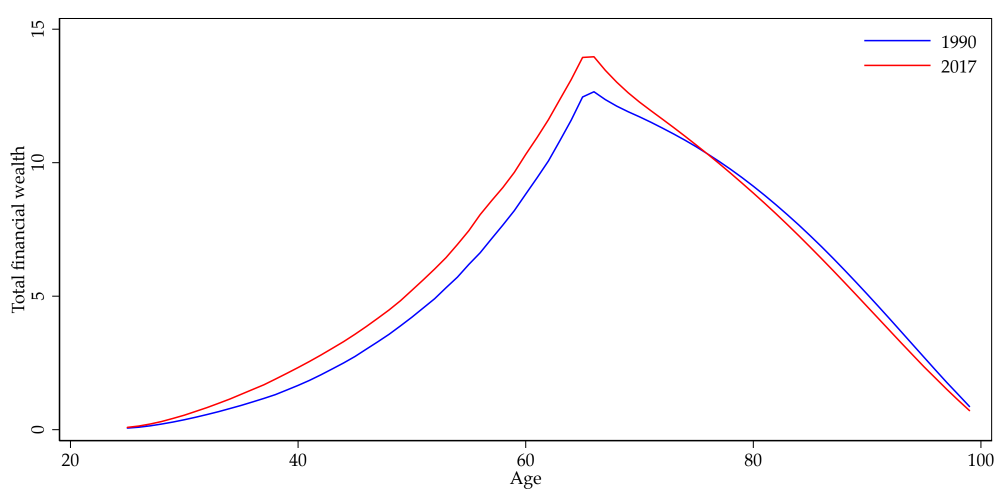


Figure 9: *Financial wealth in the model: equity and bond holdings by age*

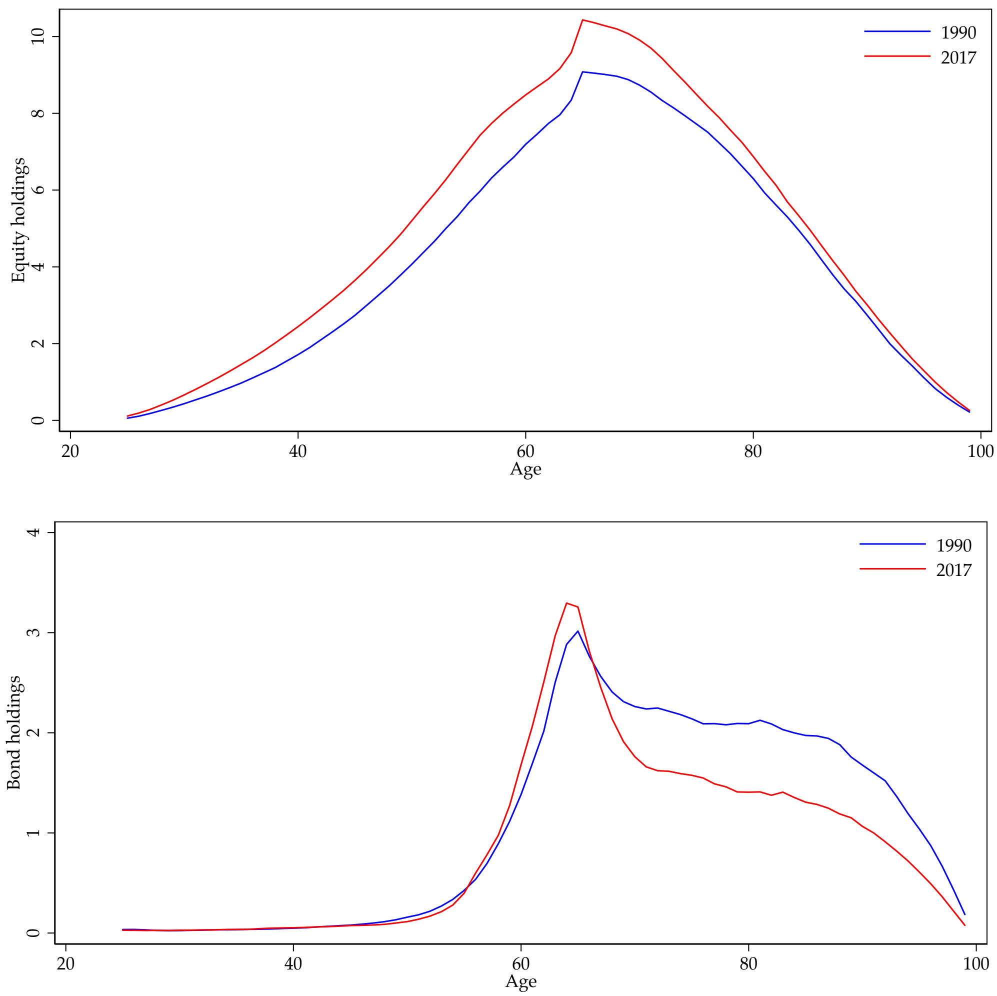


Figure 10: *Financial wealth in the model: equity share wealth by age*

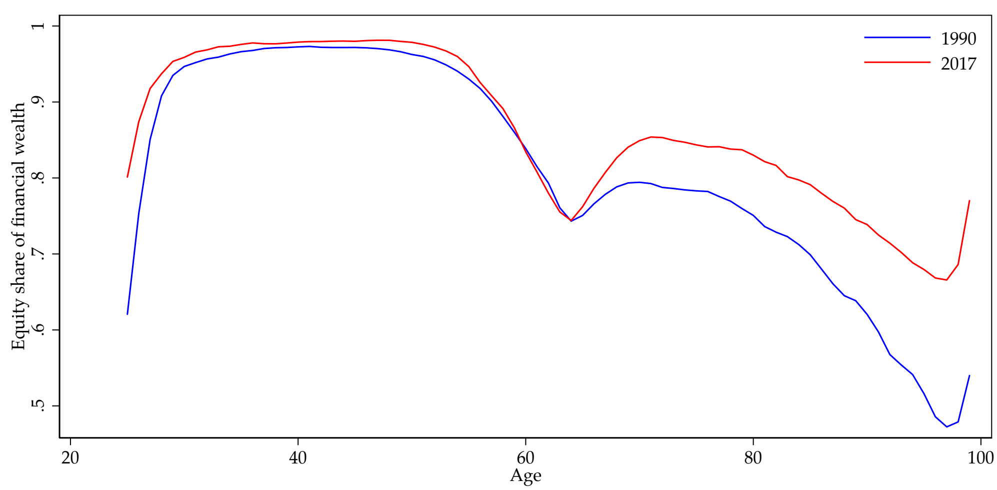


Table 2: *Returns and risk premiums in the model*

		Model				
		1970	1990	2010	2017	2050 (projected)
Equity return, mean	$\bar{r}_e$	6.94%	7.11%	5.07%	4.26%	2.20 %
s.d.	σ_e	15.22%	15.33%	15.17%	15.15%	15.12 %
Safe return, mean	$\bar{r}_f$	2.90%	4.53%	0.5%	-0.58%	-2.78%
s.d.	σ_f	4.80%	3.90%	4.92%	5.05%	4.59%
ERP	$\overline{rp}$	4.05	2.58	4.57	4.84	4.98

assets are consumed. As with all of our financial wealth dynamics, these patters are strongly dominated by the type B agents who are endogenously the only (with brief exception) holders of equity, and who have considerably more asset wealth than type A agents. This is extremely high in early years as individuals have relatively small wealth and are accumulating large amounts of equity. This is a common drawback of such models where we capture counterfactually high equity market holdings for participants.

This share falls somewhat dramatically as retirement approaches with a brief reversal driven by the peak in bond holdings seen in Figure 10. After closer inspection this seems to be due to the resolution of labor income uncertainty, which becomes a certain social security payment upon retirement. All precautionary savings motives against labor income risk disappear and households reallocate accordingly. After this households resume drawing down their share of equity, though at a faster rate in 1990. In 2017 this is more pronounced due to both general equilibrium forces discussed above and the fact that lower mortality increases the net present value of this stream of social security payments substantially making a household's average future income relatively "safer". Once again this would work against our expected result encouraging a relatively slower draw-down of risky asset holdings. If we saw this share fixed at 1990s level our effects on the ERP and safe rates would be amplified.

In Table 2 and Figures 11, we present the baseline results from our model simulations. The figures report mean levels, and table reports the mean and standard deviation of equity return and the risk free rate in each simulated economy under four past demographic conditions and also under projected population demographics in 2050. In Figure 12 we compare the model output to the data. Recall that the aim is for the model to match the U.S. data on returns and risk premiums seen earlier. The model is quite successful, and also makes some interesting predictions based on UN forecast demographic trends out to 2050.

The model clearly replicates the U-shaped path of the equity risk premium seen in the data. From a level of 405 bps in the 1970 calibration, the ERP falls to 258 bps under 1990

Figure 11: *Returns and risk premiums in the model*

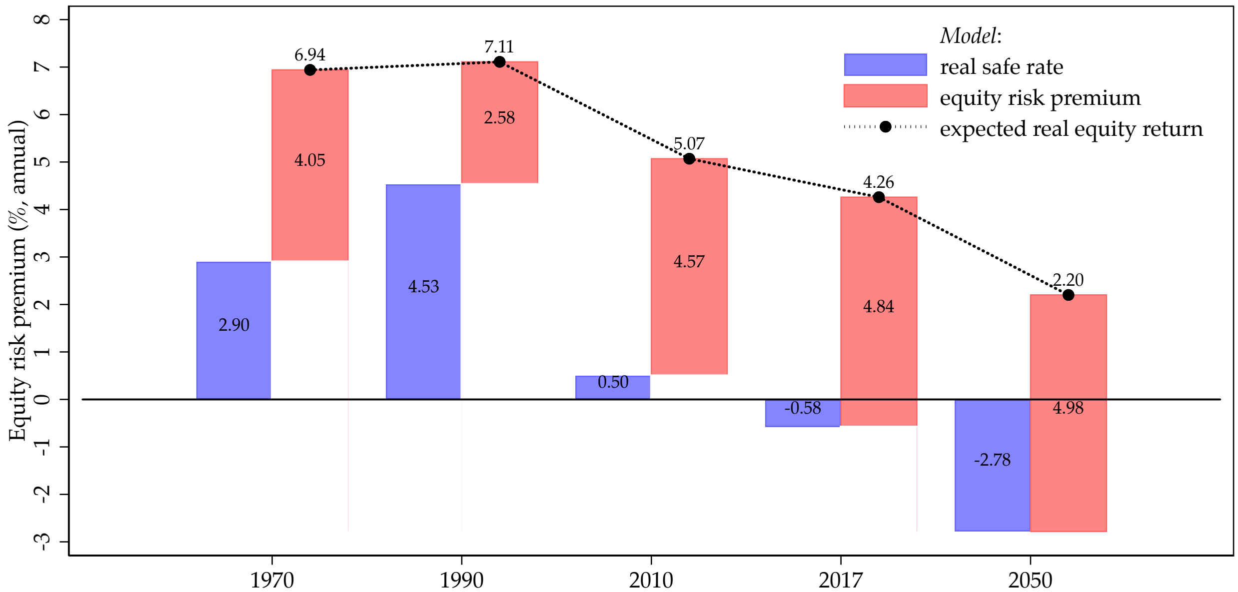
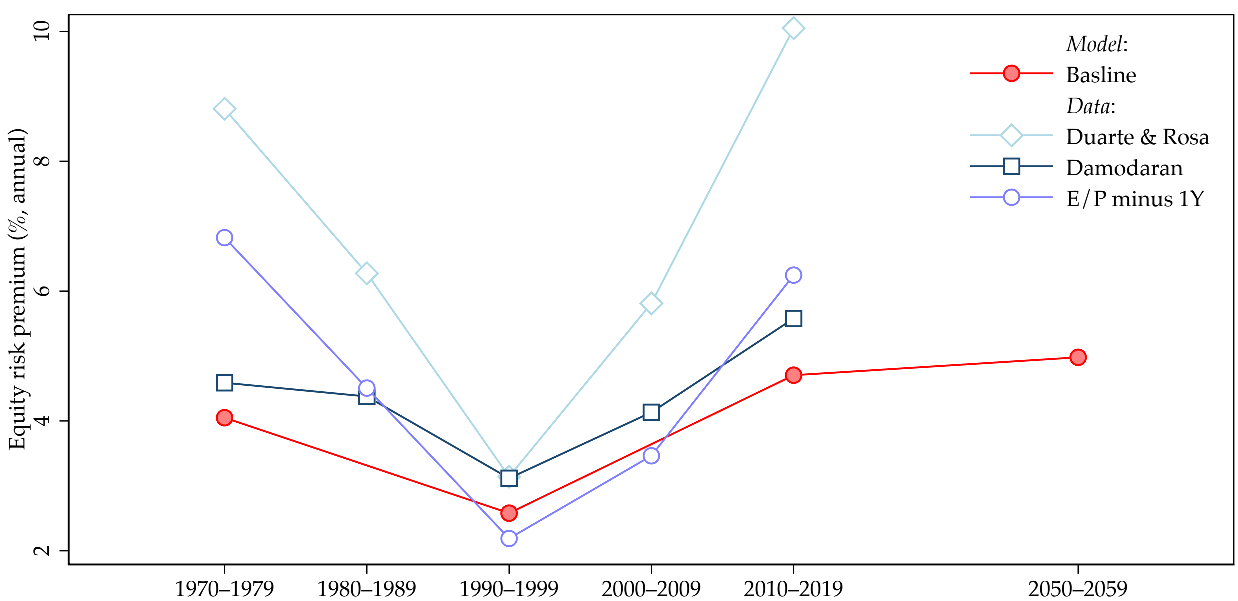


Figure 12: *Risk premiums in the model versus data*



demographics, then rises to 484 bps in the 2017 model. In 1970 the model's safe real rate is 290 bps, rising to 453 bps in 1990. But after 1990 our baseline calibration then finds a large decline in the equilibrium safe real rate, with even a negative value of –58 bps under 2017 demographics.

This decline in equilibrium safe interest rates is substantially higher than is found in much of the research documenting the effect of demographics and secular stagnation on equilibrium interest rates, suggesting that including a richer pool of assets may be important for fully understanding the implications for that question. We also find a downward pressure on the expected return to equity, but it is substantially less than that on safe interest rates.

Summing up, Figure 12 shows that between 1990 and 2017 our model can account for: roughly all of the fall and rise in the ERP "low" amplitude estimate (Damodaran); roughly half of the fall and rise in the ERP "mid" amplitude estimate (our estimate using the E/P ratio and 1-year yields); or roughly one third of the fall and rise in the ERP "high" amplitude estimate (Duarte and Rosa).

Looking to the future, the model predicts a continued fall in the risk free rate to –278 bps by 2050 with the ERP roughly stable at 498 bps. The expected return to equity therefore continues to fall to just 220 bps as demographic shifts continue to relentlessly boost the demand for all assets, but espeically safe assets.

We emphasize that these results represent steady state values calculated as if the empirical demographics in each year were permanent. Calculating the transition path of our model might yield slightly different results at any point in time, but the overall picture would likely be similar and such an exercise would represent a challenge that is computationally extreme for what we believe would be little added insight.

5.1. Model without aggregate risk

Our results are quite striking, particularly in the context of a large and growing literature on falling natural rates which finds much smaller impacts than ours for safe rates. To draw a comparison between our findings and that literature, we show the impact that shutting down aggregate risk has in our model. To do this, we eliminate the safe asset, $B^S = 0$, and remove all risk in capital markets. Most of the existing literature documenting a demographic channel for safe rates study some comparable measure of r^*. Here, we find an estimate for r^* of 4.87 under 1990 demographics and 2.13 in 2017. This change of 213 basis points over the period is much closer to the literature. In Table 3 we show our baseline results as well as results from this "risk free" change relative to some of the existing

Table 3: *Falling safe real rates: model and literature versus data*

	Period	Change in real safe rate
Model		
Baseline model: safe rate, $\bar{r}_f$	1990–2017	-5.11
Risk free model: natural rate, $r^* = \bar{r}_e$	1990–2017	-2.13
Gagnon, Johannsen, and Lopez-Salido (2016)	1980–2016	-1.25
Carvalho, Ferrero, and Nechio (2016)	1990–2014	≈ -2 *
Lisack, Sajedi, and Thwaites (2017)	1980–2015	-1.60
Eggertsson, Mehrotra, and Robbins (2019b)†	1970–2015	-4.02
Summers and Rachel (2019)‡	1970–2019	-1.70
Data		
Rachel and Smith (2015)	1990–2015	-4.50

Notes: See text. *Measure that includes social security. †Their transition dynamics show much of this fall happening from the late 1980s/early 1990s. ‡They find a 700 basis point decline in the "private" neutral rate as counterbalancing public programs have offset much of the demographic declines.

literature.[4] We report the period studied as well as the baseline effect unless otherwise noted. In general most studies find steeper declines post-1990, in line with our results.

This exercise tells us two things. The first is that our calibration has not relied on any extreme parametric choices relative to the literature to generate such dramatic results. Our decline is of a similar magnitude, and by our reading fits in sensibly when doing an apples-to-apples comparison with similar work. The second is that the introduction of both risky and safe assets to this class of models can have qualitatively large impacts on the magnitude of declines in safe rates.

This latter point is mentioned in Eggertsson, Mehrotra, and Robbins (2019b) who discuss it as one possible response to the claim that safe rates cannot be negative in equilibrium.[5] Abel, Mankiw, Summers, and Zeckhauser (1989) show that a safe interest rate below the rate of economic growth is not sufficient in creating dynamic inefficiency. The inclusion aggregate risk can lead to an equilibrium negative risk-free interest rate as it does in our benchmark model. By including aggregate risks along with the safe rate we are able to open this channel for negative interest rates that is not possible for Gagnon, Johannsen, and Lopez-Salido (2016). Our work suggests that not only are demographics quantitatively important for understanding the risk premium, but that accounting for risk will be a crucial part of understanding the role that demographics may play in secular stagnation and the long run trend in the natural rate.

[4]By "risk free" we mean that there is still idiosyncratic risk, on labor earnings in this model as well as mortality risk, just no aggregate uncertainty.

[5]The other, which their paper addresses directly, is monopoly rents

Table 4: *Results under changing Type-B preferences*

		$\rho^B = 5$			$\rho^B = 10$		
		$\psi^B = 0.2$	$\psi^B = 0.3$	$\psi^B = 0.4$	$\psi^B = 0.2$	$\psi^B = 0.3$	$\psi^B = 0.4$
1990							
Equity return, mean	$\bar{r}_e$	12.21%	7.11%	5.67%	7.44%	5.39%	4.81%
Safe return, mean	$\bar{r}_f$	9.75%	4.53%	2.01%	2.78%	1.30%	-0.40%
ERP	$\overline{rp}$	2.46	2.58	3.66	4.66	4.09	5.21
2017							
Equity return, mean	$\bar{r}_e$	5.94%	4.26%	3.75%	3.35%	3.32%	3.09%
Safe return, mean	$\bar{r}_f$	1.65%	-0.58%	-1.18%	-2.88%	-2.86%	-3.17%
ERP	$\overline{rp}$	4.29	4.84	4.93	6.23	6.18	6.26
ΔERP (1990–2017)		1.83	2.26	1.27	1.57	2.09	1.05

Notes: All other parameters identical to baseline specification, in particular those of type-A agents are: $\rho^A = 1.1$ and $\psi^A = 0.0833$.

5.2. Results under different preference parameters

The two preference parameters for each agent type have strong impact on model results and if changed can generate larger/smaller equity risk premia under any calibration. We show that the primary results of our model remain under a range of preference specifications, with the effects of demographics quantitatively large. As long as type-A agents endogenously choose to remain out of equity markets, changing their preference parameters has little impact on these results. For this reason we keep these fixed at their baseline levels, $\rho^A = 1.1$ and $\psi^A = 0.0833$, though there are a wide range of specifications that could allow a limited participation equilibrium, and their participation in equity markets would not likely reverse the effect of demographics.

In Table 4 we estimate this model under six different preferences for stock market participants, with the second column being our baseline specification. We report the for equity return, the risk free rate, and their difference (the equity risk premium) under both 1990 demographics and 2017 demographics with the difference between the two risk premia in the final row. These show that while these preferences have a large effect on the relative return on equity and risk free bonds in these models, the effect that changing demographics has on both risk free rates and the equity risk premium is consistent within some reasonable parameters. One caveat will be if we were to lower the EIS further to the point where all agents endogenously choose to not engage in equity markets, something that would happen if we lower type-B EIS to around 0.1 while keeping all other model parameters constant.

6. Conclusion

Population aging has a role to play in explaining a large number of long run macroeconomic trends. Daly (2016) suggests that global equity risk premium has risen by 2.5 percentage points since the year 2000. Similar trends can be observed in the United States, with an equity premium in 2015 that is between two and five percentage points higher than it was in the late 1990s (pre-tech bubble crisis).

In the face of this evidence for a rising equity risk premium, we show that there is a plausible demographic channel that may be driving this trend. Aging households, the boomers, are responsible for a savings glut which has helped to drive down real interest rates on safe assets, but they have relatively lower shares of their portfolio in equity. As a result, the shifting population weight moving toward these households does not have the same effect on equity returns that it has been shown to have on risk free assets.

While our work takes a simplified approach to the role of aging, the results suggest that studying the demographic channel is a fruitful approach to understanding the drivers of long-run trends in the equity premium. Not only can aging operate through the effect of changing cohort sizes, as with the aging of the baby boomer generation in the United States, but also through rising life expectancies, which can decrease individuals' willingness to take on risky assets as they age, an amplifying mechanism.

It is crucial to better understand the extent and channels through which demographics affect asset prices since the advanced economies will continue to face aging populations for the foreseeable future, and to study what, if any, policy options may be placed on the table.

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